

# What's in Main

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## Abstract

This document lists the main types, functions and syntax provided by theory *Main*. It is meant as a quick overview of what is available. For infix operators and their precedences see the final section. The sophisticated class structure is only hinted at. For details see <https://isabelle.in.tum.de/library/HOL/HOL>.

## HOL

The basic logic:  $x = y$ , *True*, *False*,  $\neg P$ ,  $P \wedge Q$ ,  $P \vee Q$ ,  $P \longrightarrow Q$ ,  $\forall x. P$ ,  $\exists x. P$ ,  $\exists!x. P$ , *THE*  $x. P$ .

*undefined* :: 'a

*default* :: 'a

## Syntax

$x \neq y$   $\equiv$   $\neg (x = y)$  ( $\neq$ )

$P \longleftrightarrow Q$   $\equiv$   $P = Q$

*if x then y else z*  $\equiv$  *If x y z*

*let x = e<sub>1</sub> in e<sub>2</sub>*  $\equiv$  *Let e<sub>1</sub> ( $\lambda x. e_2$ )*

## Orderings

A collection of classes defining basic orderings: preorder, partial order, linear order, dense linear order and wellorder.

$(\leq) \quad :: 'a \Rightarrow 'a \Rightarrow bool \quad (<=)$   
 $(<) \quad :: 'a \Rightarrow 'a \Rightarrow bool$   
 $Least \quad :: ('a \Rightarrow bool) \Rightarrow 'a$   
 $Greatest \quad :: ('a \Rightarrow bool) \Rightarrow 'a$   
 $min \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $max \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $top \quad :: 'a$   
 $bot \quad :: 'a$

### Syntax

$x \geq y \quad \equiv \quad y \leq x \quad (>=)$   
 $x > y \quad \equiv \quad y < x$   
 $\forall x \leq y. P \quad \equiv \quad \forall x. x \leq y \longrightarrow P$   
 $\exists x \leq y. P \quad \equiv \quad \exists x. x \leq y \wedge P$   
 Similarly for  $<$ ,  $\geq$  and  $>$   
 $LEAST x. P \quad \equiv \quad Least (\lambda x. P)$   
 $GREATEST x. P \quad \equiv \quad Greatest (\lambda x. P)$

## Lattices

Classes semilattice, lattice, distributive lattice and complete lattice (the latter in theory *HOL.Set*).

$inf \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $sup \quad :: 'a \Rightarrow 'a \Rightarrow 'a$   
 $Inf \quad :: 'a \text{ set} \Rightarrow 'a$   
 $Sup \quad :: 'a \text{ set} \Rightarrow 'a$

### Syntax

Available via **unbundle** *lattice\_syntax*.

$x \sqsubseteq y \quad \equiv \quad x \leq y$   
 $x \sqsubset y \quad \equiv \quad x < y$   
 $x \sqcap y \quad \equiv \quad inf \ x \ y$   
 $x \sqcup y \quad \equiv \quad sup \ x \ y$   
 $\bigsqcap A \quad \equiv \quad Inf \ A$   
 $\bigsqcup A \quad \equiv \quad Sup \ A$   
 $\top \quad \equiv \quad top$   
 $\perp \quad \equiv \quad bot$

## Set

$\{\}$	$:: 'a \text{ set}$	
$insert$	$:: 'a \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set}$	
$Collect$	$:: ('a \Rightarrow bool) \Rightarrow 'a \text{ set}$	
$(\in)$	$:: 'a \Rightarrow 'a \text{ set} \Rightarrow bool$	$(:)$
$(\cup)$	$:: 'a \text{ set} \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set}$	$(Un)$
$(\cap)$	$:: 'a \text{ set} \Rightarrow 'a \text{ set} \Rightarrow 'a \text{ set}$	$(Int)$
$\bigcup$	$:: 'a \text{ set set} \Rightarrow 'a \text{ set}$	
$\bigcap$	$:: 'a \text{ set set} \Rightarrow 'a \text{ set}$	
$Pow$	$:: 'a \text{ set} \Rightarrow 'a \text{ set set}$	
$UNIV$	$:: 'a \text{ set}$	
$(')$	$:: ('a \Rightarrow 'b) \Rightarrow 'a \text{ set} \Rightarrow 'b \text{ set}$	
$Ball$	$:: 'a \text{ set} \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	
$Bex$	$:: 'a \text{ set} \Rightarrow ('a \Rightarrow bool) \Rightarrow bool$	

## Syntax

$\{a_1, \dots, a_n\}$	$\equiv insert\ a_1\ (\dots\ (insert\ a_n\ \{\})\dots)$	
$a \notin A$	$\equiv \neg(x \in A)$	
$A \subseteq B$	$\equiv A \leq B$	
$A \subset B$	$\equiv A < B$	
$A \supseteq B$	$\equiv B \leq A$	
$A \supset B$	$\equiv B < A$	
$\{x. P\}$	$\equiv Collect\ (\lambda x. P)$	
$\{t \mid x_1 \dots x_n. P\}$	$\equiv \{v. \exists x_1 \dots x_n. v = t \wedge P\}$	
$\bigcup_{x \in I}. A$	$\equiv \bigcup ((\lambda x. A) ' I)$	$(UN)$
$\bigcup x. A$	$\equiv \bigcup ((\lambda x. A) ' UNIV)$	
$\bigcap_{x \in I}. A$	$\equiv \bigcap ((\lambda x. A) ' I)$	$(INT)$
$\bigcap x. A$	$\equiv \bigcap ((\lambda x. A) ' UNIV)$	
$\forall x \in A. P$	$\equiv Ball\ A\ (\lambda x. P)$	
$\exists x \in A. P$	$\equiv Bex\ A\ (\lambda x. P)$	
$range\ f$	$\equiv f ' UNIV$	

## Fun

<i>id</i>	:: 'a ⇒ 'a	
(○)	:: ('a ⇒ 'b) ⇒ ('c ⇒ 'a) ⇒ 'c ⇒ 'b	(○)
<i>inj_on</i>	:: ('a ⇒ 'b) ⇒ 'a set ⇒ bool	
<i>inj</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>surj</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>bij</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>bij_betw</i>	:: ('a ⇒ 'b) ⇒ 'a set ⇒ 'b set ⇒ bool	
<i>monotone_on</i>	:: 'a set ⇒ ('a ⇒ 'a ⇒ bool) ⇒ ('b ⇒ 'b ⇒ bool) ⇒ ('a ⇒ 'b) ⇒ bool	
<i>monotone</i>	:: ('a ⇒ 'a ⇒ bool) ⇒ ('b ⇒ 'b ⇒ bool) ⇒ ('a ⇒ 'b) ⇒ bool	
<i>mono_on</i>	:: 'a set ⇒ ('a ⇒ 'b) ⇒ bool	
<i>mono</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>strict_mono_on</i>	:: 'a set ⇒ ('a ⇒ 'b) ⇒ bool	
<i>strict_mono</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>antimono</i>	:: ('a ⇒ 'b) ⇒ bool	
<i>fun_upd</i>	:: ('a ⇒ 'b) ⇒ 'a ⇒ 'b ⇒ 'a ⇒ 'b	

## Syntax

$f(x := y)$	≡	$fun\_upd\ f\ x\ y$
$f(x_1:=y_1, \dots, x_n:=y_n)$	≡	$f(x_1:=y_1) \dots (x_n:=y_n)$

## Hilbert\_\_Choice

Hilbert's selection ( $\varepsilon$ ) operator: *SOME*  $x$ .  $P$ .

$inv\_into :: 'a\ set \Rightarrow ('a \Rightarrow 'b) \Rightarrow 'b \Rightarrow 'a$

## Syntax

$inv \equiv inv\_into\ UNIV$

## Fixed Points

Theory: *HOL.Inductive*.

Least and greatest fixed points in a complete lattice 'a:

$lfp :: ('a \Rightarrow 'a) \Rightarrow 'a$

$gfp :: ('a \Rightarrow 'a) \Rightarrow 'a$

Note that in particular sets ( $'a \Rightarrow bool$ ) are complete lattices.

## Sum\_Type

Type constructor  $+$ .

$Inl$   $:: 'a \Rightarrow 'a + 'b$

$Inr$   $:: 'a \Rightarrow 'b + 'a$

$(<+>)$   $:: 'a \text{ set} \Rightarrow 'b \text{ set} \Rightarrow ('a + 'b) \text{ set}$

## Product\_Type

Types  $unit$  and  $\times$ .

$()$   $:: unit$

$Pair$   $:: 'a \Rightarrow 'b \Rightarrow 'a \times 'b$

$fst$   $:: 'a \times 'b \Rightarrow 'a$

$snd$   $:: 'a \times 'b \Rightarrow 'b$

$case\_prod$   $:: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a \times 'b \Rightarrow 'c$

$curry$   $:: ('a \times 'b \Rightarrow 'c) \Rightarrow 'a \Rightarrow 'b \Rightarrow 'c$

$Sigma$   $:: 'a \text{ set} \Rightarrow ('a \Rightarrow 'b \text{ set}) \Rightarrow ('a \times 'b) \text{ set}$

### Syntax

$(a, b)$   $\equiv Pair\ a\ b$

$\lambda(x, y). t$   $\equiv case\_prod\ (\lambda x\ y. t)$

$A \times B$   $\equiv Sigma\ A\ (\lambda_. B)$

Pairs may be nested. Nesting to the right is printed as a tuple, e.g.  $(a, b, c)$  is really  $(a, (b, c))$ . Pattern matching with pairs and tuples extends to all binders, e.g.  $\forall (x, y) \in A. P$ ,  $\{(x, y). P\}$ , etc.

## Relation

$converse$   $:: ('a \times 'b) \text{ set} \Rightarrow ('b \times 'a) \text{ set}$

$(O)$   $:: ('a \times 'b) \text{ set} \Rightarrow ('b \times 'c) \text{ set} \Rightarrow ('a \times 'c) \text{ set}$

$(“)$   $:: ('a \times 'b) \text{ set} \Rightarrow 'a \text{ set} \Rightarrow 'b \text{ set}$

$inv\_image$   $:: ('a \times 'a) \text{ set} \Rightarrow ('b \Rightarrow 'a) \Rightarrow ('b \times 'b) \text{ set}$

$Id\_on$   $:: 'a \text{ set} \Rightarrow ('a \times 'a) \text{ set}$

$Id$   $:: ('a \times 'a) \text{ set}$

$Domain$   $:: ('a \times 'b) \text{ set} \Rightarrow 'a \text{ set}$

$Range$   $:: ('a \times 'b) \text{ set} \Rightarrow 'b \text{ set}$

*Field*     :: ('a × 'a) set ⇒ 'a set  
*refl\_on*    :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*refl*        :: ('a × 'a) set ⇒ bool  
*sym*         :: ('a × 'a) set ⇒ bool  
*antisym*    :: ('a × 'a) set ⇒ bool  
*trans*       :: ('a × 'a) set ⇒ bool  
*irrefl*     :: ('a × 'a) set ⇒ bool  
*total\_on*   :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*total*      :: ('a × 'a) set ⇒ bool

## Syntax

$r^{-1} \equiv \text{converse } r \quad (\wedge^{-1})$

Type synonym  $'a \text{ rel} = ('a \times 'a) \text{ set}$

## Equiv\_Relations

*equiv*        :: 'a set ⇒ ('a × 'a) set ⇒ bool  
*(//)*         :: 'a set ⇒ ('a × 'a) set ⇒ 'a set set  
*congruent*    :: ('a × 'a) set ⇒ ('a ⇒ 'b) ⇒ bool  
*congruent2* :: ('a × 'a) set ⇒ ('b × 'b) set ⇒ ('a ⇒ 'b ⇒ 'c) ⇒ bool

## Syntax

*f respects r*    ≡ *congruent r f*  
*f respects2 r*  ≡ *congruent2 r r f*

## Transitive\_Closure

*rtrancl* :: ('a × 'a) set ⇒ ('a × 'a) set  
*trancl*  :: ('a × 'a) set ⇒ ('a × 'a) set  
*reflcl*  :: ('a × 'a) set ⇒ ('a × 'a) set  
*acyclic* :: ('a × 'a) set ⇒ bool  
*( $\widetilde{\sim}$ )*    :: ('a × 'a) set ⇒ nat ⇒ ('a × 'a) set

## Syntax

$r^* \equiv rtrancl\ r \quad (\hat{~}^*)$   
 $r^+ \equiv trancl\ r \quad (\hat{~}^+)$   
 $r^- \equiv reflcl\ r \quad (\hat{~}^-)$

## Algebra

Theories *HOL.Groups*, *HOL.Rings*, *HOL.Fields* and *HOL.Divides* define a large collection of classes describing common algebraic structures from semi-groups up to fields. Everything is done in terms of overloaded operators:

$0 \quad \quad \quad :: 'a$   
 $1 \quad \quad \quad :: 'a$   
 $(+)$      $:: 'a \Rightarrow 'a \Rightarrow 'a$   
 $(-)$      $:: 'a \Rightarrow 'a \Rightarrow 'a$   
*uminus*  $:: 'a \Rightarrow 'a \quad \quad \quad (-)$   
 $(*)$      $:: 'a \Rightarrow 'a \Rightarrow 'a$   
*inverse*  $:: 'a \Rightarrow 'a$   
 $(div)$     $:: 'a \Rightarrow 'a \Rightarrow 'a$   
*abs*     $:: 'a \Rightarrow 'a$   
*sgn*     $:: 'a \Rightarrow 'a$   
 $(dvd)$     $:: 'a \Rightarrow 'a \Rightarrow bool$   
 $(div)$     $:: 'a \Rightarrow 'a \Rightarrow 'a$   
 $(mod)$     $:: 'a \Rightarrow 'a \Rightarrow 'a$

## Syntax

$|x| \equiv abs\ x$

## Nat

**datatype** *nat* = 0 | *Suc nat*

$(+)$     $(-)$     $(*)$     $(\hat{~})$     $(div)$     $(mod)$     $(dvd)$   
 $(\leq)$     $(<)$    *min*   *max*   *Min*   *Max*  
*of\_nat*  $:: nat \Rightarrow 'a$   
 $(\hat{~})$      $:: ('a \Rightarrow 'a) \Rightarrow nat \Rightarrow 'a \Rightarrow 'a$

## Int

Type *int*

(+) (-) *uminus* (\*) ( $\wedge$ ) (*div*) (*mod*) (*dvd*)  
( $\leq$ ) (<) *min* *max* *Min* *Max*  
*abs* *sgn*

*nat* :: *int*  $\Rightarrow$  *nat*

*of\_int* :: *int*  $\Rightarrow$  'a

$\mathbb{Z}$  :: 'a *set* (Ints)

### Syntax

*int*  $\equiv$  *of\_nat*

## Finite\_Set

*finite* :: 'a *set*  $\Rightarrow$  *bool*

*card* :: 'a *set*  $\Rightarrow$  *nat*

*Finite\_Set.fold* :: ('a  $\Rightarrow$  'b  $\Rightarrow$  'b)  $\Rightarrow$  'b  $\Rightarrow$  'a *set*  $\Rightarrow$  'b

## Lattices\_Big

*Min* :: 'a *set*  $\Rightarrow$  'a

*Max* :: 'a *set*  $\Rightarrow$  'a

*arg\_min* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  ('a  $\Rightarrow$  *bool*)  $\Rightarrow$  'a

*is\_arg\_min* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  ('a  $\Rightarrow$  *bool*)  $\Rightarrow$  'a  $\Rightarrow$  *bool*

*arg\_max* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  ('a  $\Rightarrow$  *bool*)  $\Rightarrow$  'a

*is\_arg\_max* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  ('a  $\Rightarrow$  *bool*)  $\Rightarrow$  'a  $\Rightarrow$  *bool*

### Syntax

*ARG\_MIN* *f x. P*  $\equiv$  *arg\_min* *f* ( $\lambda x. P$ )

*ARG\_MAX* *f x. P*  $\equiv$  *arg\_max* *f* ( $\lambda x. P$ )

## Groups\_Big

*sum* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  'a *set*  $\Rightarrow$  'b

*prod* :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  'a *set*  $\Rightarrow$  'b



## Syntax

$$\begin{aligned}\sum A &\equiv \text{sum } (\lambda x. x) A \quad (\text{SUM}) \\ \sum_{x \in A}. t &\equiv \text{sum } (\lambda x. t) A \\ \sum x | P. t &\equiv \sum x | P. t \\ \text{Similarly for } \prod &\text{ instead of } \sum \quad (\text{PROD})\end{aligned}$$

## Wellfounded

$$\begin{aligned}\text{wf} &:: ('a \times 'a) \text{ set} \Rightarrow \text{bool} \\ \text{Wellfounded.acc} &:: ('a \times 'a) \text{ set} \Rightarrow 'a \text{ set} \\ \text{measure} &:: ('a \Rightarrow \text{nat}) \Rightarrow ('a \times 'a) \text{ set} \\ (<*\text{lex}* >) &:: ('a \times 'a) \text{ set} \Rightarrow ('b \times 'b) \text{ set} \Rightarrow (('a \times 'b) \times 'a \times 'b) \text{ set} \\ (<*\text{mlex}* >) &:: ('a \Rightarrow \text{nat}) \Rightarrow ('a \times 'a) \text{ set} \Rightarrow ('a \times 'a) \text{ set} \\ \text{less\_than} &:: (\text{nat} \times \text{nat}) \text{ set} \\ \text{pred\_nat} &:: (\text{nat} \times \text{nat}) \text{ set}\end{aligned}$$

## Set\_Interval

$$\begin{aligned}\text{lessThan} &:: 'a \Rightarrow 'a \text{ set} \\ \text{atMost} &:: 'a \Rightarrow 'a \text{ set} \\ \text{greaterThan} &:: 'a \Rightarrow 'a \text{ set} \\ \text{atLeast} &:: 'a \Rightarrow 'a \text{ set} \\ \text{greaterThanLessThan} &:: 'a \Rightarrow 'a \Rightarrow 'a \text{ set} \\ \text{atLeastLessThan} &:: 'a \Rightarrow 'a \Rightarrow 'a \text{ set} \\ \text{greaterThanAtMost} &:: 'a \Rightarrow 'a \Rightarrow 'a \text{ set} \\ \text{atLeastAtMost} &:: 'a \Rightarrow 'a \Rightarrow 'a \text{ set}\end{aligned}$$

## Syntax

$\{..<y\}$	$\equiv$	$lessThan\ y$
$\{..y\}$	$\equiv$	$atMost\ y$
$\{x<..\}$	$\equiv$	$greaterThan\ x$
$\{x..\}$	$\equiv$	$atLeast\ x$
$\{x<..<y\}$	$\equiv$	$greaterThanLessThan\ x\ y$
$\{x..<y\}$	$\equiv$	$atLeastLessThan\ x\ y$
$\{x<..y\}$	$\equiv$	$greaterThanAtMost\ x\ y$
$\{x..y\}$	$\equiv$	$atLeastAtMost\ x\ y$
$\bigcup_{i \leq n}. A$	$\equiv$	$\bigcup_{i \in \{..n\}}. A$
$\bigcup_{i < n}. A$	$\equiv$	$\bigcup_{i \in \{..<n\}}. A$

Similarly for  $\bigcap$  instead of  $\bigcup$

$\sum x = a..b. t$	$\equiv$	$sum\ (\lambda x. t)\ \{a..b\}$
$\sum x = a..<b. t$	$\equiv$	$sum\ (\lambda x. t)\ \{a..<b\}$
$\sum x \leq b. t$	$\equiv$	$sum\ (\lambda x. t)\ \{..b\}$
$\sum x < b. t$	$\equiv$	$sum\ (\lambda x. t)\ \{..<b\}$

Similarly for  $\prod$  instead of  $\sum$

## Power

$(\wedge) :: 'a \Rightarrow nat \Rightarrow 'a$

## Option

**datatype**  $'a\ option = None \mid Some\ 'a$

$the :: 'a\ option \Rightarrow 'a$   
 $map\_option :: ('a \Rightarrow 'b) \Rightarrow 'a\ option \Rightarrow 'b\ option$   
 $set\_option :: 'a\ option \Rightarrow 'a\ set$   
 $Option.bind :: 'a\ option \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'b\ option$

## List

**datatype**  $'a\ list = [] \mid (\#)\ 'a\ ('a\ list)$

$(@) :: 'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$

*butlast*     :: 'a list  $\Rightarrow$  'a list  
*concat*     :: 'a list list  $\Rightarrow$  'a list  
*distinct*    :: 'a list  $\Rightarrow$  bool  
*drop*        :: nat  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*dropWhile* :: ('a  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*filter*     :: ('a  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*find*        :: ('a  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'a option  
*fold*        :: ('a  $\Rightarrow$  'b  $\Rightarrow$  'b)  $\Rightarrow$  'a list  $\Rightarrow$  'b  $\Rightarrow$  'b  
*foldr*     :: ('a  $\Rightarrow$  'b  $\Rightarrow$  'b)  $\Rightarrow$  'a list  $\Rightarrow$  'b  $\Rightarrow$  'b  
*foldl*     :: ('a  $\Rightarrow$  'b  $\Rightarrow$  'a)  $\Rightarrow$  'a  $\Rightarrow$  'b list  $\Rightarrow$  'a  
*hd*         :: 'a list  $\Rightarrow$  'a  
*last*        :: 'a list  $\Rightarrow$  'a  
*length*     :: 'a list  $\Rightarrow$  nat  
*lenlex*     :: ('a  $\times$  'a) set  $\Rightarrow$  ('a list  $\times$  'a list) set  
*lex*         :: ('a  $\times$  'a) set  $\Rightarrow$  ('a list  $\times$  'a list) set  
*lexn*        :: ('a  $\times$  'a) set  $\Rightarrow$  nat  $\Rightarrow$  ('a list  $\times$  'a list) set  
*lexord*     :: ('a  $\times$  'a) set  $\Rightarrow$  ('a list  $\times$  'a list) set  
*listrel*    :: ('a  $\times$  'b) set  $\Rightarrow$  ('a list  $\times$  'b list) set  
*listrel1*   :: ('a  $\times$  'a) set  $\Rightarrow$  ('a list  $\times$  'a list) set  
*lists*      :: 'a set  $\Rightarrow$  'a list set  
*listset*    :: 'a set list  $\Rightarrow$  'a list set  
*list\_all2*  :: ('a  $\Rightarrow$  'b  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'b list  $\Rightarrow$  bool  
*list\_update*:: 'a list  $\Rightarrow$  nat  $\Rightarrow$  'a  $\Rightarrow$  'a list  
*map*         :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  'a list  $\Rightarrow$  'b list  
*measures*   :: ('a  $\Rightarrow$  nat) list  $\Rightarrow$  ('a  $\times$  'a) set  
*(!)*         :: 'a list  $\Rightarrow$  nat  $\Rightarrow$  'a  
*nths*        :: 'a list  $\Rightarrow$  nat set  $\Rightarrow$  'a list  
*prod\_list*  :: 'a list  $\Rightarrow$  'a  
*remdups*    :: 'a list  $\Rightarrow$  'a list  
*removeAll*  :: 'a  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*remove1*    :: 'a  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*replicate*  :: nat  $\Rightarrow$  'a  $\Rightarrow$  'a list  
*rev*         :: 'a list  $\Rightarrow$  'a list  
*rotate*     :: nat  $\Rightarrow$  'a list  $\Rightarrow$  'a list  
*rotate1*    :: 'a list  $\Rightarrow$  'a list  
*set*         :: 'a list  $\Rightarrow$  'a set  
*shuffles*   :: 'a list  $\Rightarrow$  'a list  $\Rightarrow$  'a list set  
*sort*        :: 'a list  $\Rightarrow$  'a list  
*sorted*     :: 'a list  $\Rightarrow$  bool

$sorted\_wrt :: ('a \Rightarrow 'a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow bool$   
 $splice :: 'a\ list \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $sum\_list :: 'a\ list \Rightarrow 'a$   
 $take :: nat \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $takeWhile :: ('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ list$   
 $tl :: 'a\ list \Rightarrow 'a\ list$   
 $upt :: nat \Rightarrow nat \Rightarrow nat\ list$   
 $upto :: int \Rightarrow int \Rightarrow int\ list$   
 $zip :: 'a\ list \Rightarrow 'b\ list \Rightarrow ('a \times 'b)\ list$

## Syntax

$[x_1, \dots, x_n] \equiv x_1 \# \dots \# x_n \# []$   
 $[m..<n] \equiv upt\ m\ n$   
 $[i..j] \equiv upto\ i\ j$   
 $xs[n := x] \equiv list\_update\ xs\ n\ x$   
 $\sum x \leftarrow xs. e \equiv listsum\ (map\ (\lambda x. e)\ xs)$

Filter input syntax  $[pat \leftarrow e. b]$ , where  $pat$  is a tuple pattern, which stands for  $filter\ (\lambda pat. b)\ e$ .

List comprehension input syntax:  $[e. q_1, \dots, q_n]$  where each qualifier  $q_i$  is either a generator  $pat \leftarrow e$  or a guard, i.e. boolean expression.

## Map

Maps model partial functions and are often used as finite tables. However, the domain of a map may be infinite.

$Map.empty :: 'a \Rightarrow 'b\ option$   
 $(++) :: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow 'a \Rightarrow 'b\ option$   
 $(\circ_m) :: ('a \Rightarrow 'b\ option) \Rightarrow ('c \Rightarrow 'a\ option) \Rightarrow 'c \Rightarrow 'b\ option$   
 $(|') :: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set \Rightarrow 'a \Rightarrow 'b\ option$   
 $dom :: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ set$   
 $ran :: ('a \Rightarrow 'b\ option) \Rightarrow 'b\ set$   
 $(\subseteq_m) :: ('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b\ option) \Rightarrow bool$   
 $map\_of :: ('a \times 'b)\ list \Rightarrow 'a \Rightarrow 'b\ option$   
 $map\_upds :: ('a \Rightarrow 'b\ option) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow 'a \Rightarrow 'b\ option$

## Syntax

$\lambda x. None$	$\equiv$	$\lambda \_ . None$
$m(x \mapsto y)$	$\equiv$	$m(x := Some\ y)$
$m(x_1 \mapsto y_1, \dots, x_n \mapsto y_n)$	$\equiv$	$m(x_1 \mapsto y_1) \dots (x_n \mapsto y_n)$
$[x_1 \mapsto y_1, \dots, x_n \mapsto y_n]$	$\equiv$	$Map.empty(x_1 \mapsto y_1, \dots, x_n \mapsto y_n)$
$m(xs \ [\mapsto] \ ys)$	$\equiv$	$map\_upds\ m\ xs\ ys$

## Infix operators in Main

	Operator	precedence	associativity
Meta-logic	$\implies$	1	right
	$\equiv$	2	
Logic	$\wedge$	35	right
	$\vee$	30	right
	$\longrightarrow, \longleftrightarrow$	25	right
	$=, \neq$	50	left
Orderings	$\leq, <, \geq, >$	50	
Sets	$\subseteq, \subset, \supseteq, \supset$	50	
	$\in, \notin$	50	
	$\cap$	70	left
	$\cup$	65	left
Functions and Relations	$\circ$	55	left
	$'$	90	right
	$O$	75	right
	$''$	90	right
	$\sim$	80	right
Numbers	$+, -$	65	left
	$*, /$	70	left
	$div, mod$	70	left
	$\wedge$	80	right
	$dvd$	50	
Lists	$\#, @$	65	right
	$!$	100	left