

State Spaces: The Locale Way

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1 Introduction

These theories introduce a new command called **statespace**. It's usage is similar to **records**. However, the command does not introduce a new type but an abstract specification based on the locale infrastructure. This leads to extra flexibility in composing state space components, in particular multiple inheritance and renaming of components.

The state space infrastructure basically manages the following things:

- distinctness of field names
- projections / injections from / to an abstract *value* type
- syntax translations for lookup and update, hiding the projections and injections
- simplification procedure for lookups / updates, similar to records

Overview In Section 2 we define distinctness of the nodes in a binary tree and provide the basic prover tools to support efficient distinctness reasoning for field names managed by state spaces. The state is represented as a function from (abstract) names to (abstract) values as introduced in Section 3. The basic setup for state spaces is in Section 4. Some syntax for lookup and updates is added in Section 5. Finally Section 6 explains the usage of state spaces by examples.

2 Distinctness of Names in a Binary Tree

```
theory DistinctTreeProver
imports Main
begin
```

A state space manages a set of (abstract) names and assumes that the names are distinct. The names are stored as parameters of a locale and distinctness as an assumption. The most common request is to proof distinctness of two given names. We maintain the names in a balanced binary tree and formulate a predicate that all nodes in the tree have distinct names. This setup leads to logarithmic certificates.

2.1 The Binary Tree

```
datatype 'a tree = Node 'a tree 'a bool 'a tree | Tip
```

The boolean flag in the node marks the content of the node as deleted, without having to build a new tree. We prefer the boolean flag to an option type, so that the ML-layer can still use the node content to facilitate binary search in the tree. The ML code keeps the nodes sorted using the term order. We do not have to push ordering to the HOL level.

2.2 Distinctness of Nodes

```
primrec set-of :: 'a tree  $\Rightarrow$  'a set
where
  set-of Tip = {}
| set-of (Node l x d r) = (if d then {} else {x})  $\cup$  set-of l  $\cup$  set-of r

primrec all-distinct :: 'a tree  $\Rightarrow$  bool
where
  all-distinct Tip = True
| all-distinct (Node l x d r) =
  ((d  $\vee$  (x  $\notin$  set-of l  $\wedge$  x  $\notin$  set-of r))  $\wedge$ 
   set-of l  $\cap$  set-of r = {}  $\wedge$ 
   all-distinct l  $\wedge$  all-distinct r)
```

Given a binary tree t for which *all-distinct* holds, given two different nodes contained in the tree, we want to write a ML function that generates a logarithmic certificate that the content of the nodes is distinct. We use the following lemmas to achieve this.

lemma *all-distinct-left*: $\text{all-distinct } (\text{Node } l \ x \ b \ r) \implies \text{all-distinct } l$
 $\langle \text{proof} \rangle$

lemma *all-distinct-right*: $\text{all-distinct } (\text{Node } l \ x \ b \ r) \implies \text{all-distinct } r$
 $\langle \text{proof} \rangle$

lemma *distinct-left*: $\text{all-distinct } (\text{Node } l \ x \ \text{False} \ r) \implies y \in \text{set-of } l \implies x \neq y$
 $\langle \text{proof} \rangle$

lemma *distinct-right*: $\text{all-distinct } (\text{Node } l \ x \ \text{False} \ r) \implies y \in \text{set-of } r \implies x \neq y$
 $\langle \text{proof} \rangle$

lemma *distinct-left-right*:
 $\text{all-distinct } (\text{Node } l \ z \ b \ r) \implies x \in \text{set-of } l \implies y \in \text{set-of } r \implies x \neq y$
 $\langle \text{proof} \rangle$

lemma *in-set-root*: $x \in \text{set-of } (\text{Node } l \ x \ \text{False} \ r)$
 $\langle \text{proof} \rangle$

lemma *in-set-left*: $y \in \text{set-of } l \implies y \in \text{set-of } (\text{Node } l \ x \ \text{False} \ r)$
 $\langle \text{proof} \rangle$

lemma *in-set-right*: $y \in \text{set-of } r \implies y \in \text{set-of } (\text{Node } l \ x \ \text{False} \ r)$
 $\langle \text{proof} \rangle$

lemma *swap-neq*: $x \neq y \implies y \neq x$
 $\langle \text{proof} \rangle$

lemma *neq-to-eq-False*: $x \neq y \implies (x=y) \equiv \text{False}$
 $\langle \text{proof} \rangle$

2.3 Containment of Trees

When deriving a state space from other ones, we create a new name tree which contains all the names of the parent state spaces and assume the predicate *all-distinct*. We then prove that the new locale interprets all parent locales. Hence we have to show that the new distinctness assumption on all names implies the distinctness assumptions of the parent locales. This proof is implemented in ML. We do this efficiently by defining a kind of containment check of trees by “subtraction”. We subtract the parent tree from the new tree. If this succeeds we know that *all-distinct* of the new tree implies *all-distinct* of the parent tree. The resulting certificate is of the order $n * \log m$ where n is the size of the (smaller) parent tree and m the

size of the (bigger) new tree.

primrec *delete* :: 'a \Rightarrow 'a tree \Rightarrow 'a tree option

where

delete *x* *Tip* = *None*
| *delete* *x* (*Node* *l* *y* *d* *r*) = (case *delete* *x* *l* of
 Some *l'* \Rightarrow
 (case *delete* *x* *r* of
 Some *r'* \Rightarrow *Some* (*Node* *l'* *y* (*d* \vee (*x*=*y*)) *r'*)
 | *None* \Rightarrow *Some* (*Node* *l'* *y* (*d* \vee (*x*=*y*)) *r*)
 | *None* \Rightarrow
 (case *delete* *x* *r* of
 Some *r'* \Rightarrow *Some* (*Node* *l* *y* (*d* \vee (*x*=*y*)) *r'*)
 | *None* \Rightarrow if *x*=*y* \wedge \neg *d* then *Some* (*Node* *l* *y* *True* *r*)
 else *None*))

lemma *delete-Some-set-of*: *delete* *x* *t* = *Some* *t'* \Longrightarrow *set-of* *t'* \subseteq *set-of* *t*
<proof>

lemma *delete-Some-all-distinct*:

delete *x* *t* = *Some* *t'* \Longrightarrow *all-distinct* *t* \Longrightarrow *all-distinct* *t'*
<proof>

lemma *delete-None-set-of-conv*: *delete* *x* *t* = *None* = (*x* \notin *set-of* *t*)
<proof>

lemma *delete-Some-x-set-of*:

delete *x* *t* = *Some* *t'* \Longrightarrow *x* \in *set-of* *t* \wedge *x* \notin *set-of* *t'*
<proof>

primrec *subtract* :: 'a tree \Rightarrow 'a tree \Rightarrow 'a tree option

where

subtract *Tip* *t* = *Some* *t*
| *subtract* (*Node* *l* *x* *b* *r*) *t* =
 (case *delete* *x* *t* of
 Some *t'* \Rightarrow (case *subtract* *l* *t'* of
 Some *t''* \Rightarrow *subtract* *r* *t''*
 | *None* \Rightarrow *None*)
 | *None* \Rightarrow *None*)

lemma *subtract-Some-set-of-res*:

subtract *t*₁ *t*₂ = *Some* *t* \Longrightarrow *set-of* *t* \subseteq *set-of* *t*₂
<proof>

lemma *subtract-Some-set-of*:

subtract *t*₁ *t*₂ = *Some* *t* \Longrightarrow *set-of* *t*₁ \subseteq *set-of* *t*₂
<proof>

lemma *subtract-Some-all-distinct-res:*
 $subtract\ t_1\ t_2 = Some\ t \implies all-distinct\ t_2 \implies all-distinct\ t$
 $\langle proof \rangle$

lemma *subtract-Some-dist-res:*
 $subtract\ t_1\ t_2 = Some\ t \implies set-of\ t_1 \cap set-of\ t = \{\}$
 $\langle proof \rangle$

lemma *subtract-Some-all-distinct:*
 $subtract\ t_1\ t_2 = Some\ t \implies all-distinct\ t_2 \implies all-distinct\ t_1$
 $\langle proof \rangle$

lemma *delete-left:*
assumes *dist:* $all-distinct\ (Node\ l\ y\ d\ r)$
assumes *del-l:* $delete\ x\ l = Some\ l'$
shows $delete\ x\ (Node\ l\ y\ d\ r) = Some\ (Node\ l'\ y\ d\ r)$
 $\langle proof \rangle$

lemma *delete-right:*
assumes *dist:* $all-distinct\ (Node\ l\ y\ d\ r)$
assumes *del-r:* $delete\ x\ r = Some\ r'$
shows $delete\ x\ (Node\ l\ y\ d\ r) = Some\ (Node\ l\ y\ d\ r')$
 $\langle proof \rangle$

lemma *delete-root:*
assumes *dist:* $all-distinct\ (Node\ l\ x\ False\ r)$
shows $delete\ x\ (Node\ l\ x\ False\ r) = Some\ (Node\ l\ x\ True\ r)$
 $\langle proof \rangle$

lemma *subtract-Node:*
assumes *del:* $delete\ x\ t = Some\ t'$
assumes *sub-l:* $subtract\ l\ t' = Some\ t''$
assumes *sub-r:* $subtract\ r\ t'' = Some\ t'''$
shows $subtract\ (Node\ l\ x\ False\ r)\ t = Some\ t'''$
 $\langle proof \rangle$

lemma *subtract-Tip:* $subtract\ Tip\ t = Some\ t$
 $\langle proof \rangle$

Now we have all the theorems in place that are needed for the certificate generating ML functions.

$\langle ML \rangle$

end

3 State Space Representation as Function

theory *StateFun* **imports** *DistinctTreeProver*
begin

The state space is represented as a function from names to values. We neither fix the type of names nor the type of values. We define lookup and update functions and provide simprocs that simplify expressions containing these, similar to HOL-records.

The lookup and update function get constructor/destructor functions as parameters. These are used to embed various HOL-types into the abstract value type. Conceptually the abstract value type is a sum of all types that we attempt to store in the state space.

The update is actually generalized to a map function. The map supplies better compositionality, especially if you think of nested state spaces.

definition *K-statefun* :: $'a \Rightarrow 'b \Rightarrow 'a$ **where** $K\text{-statefun } c \ x \equiv c$

lemma *K-statefun-apply* [*simp*]: $K\text{-statefun } c \ x = c$
 $\langle \text{proof} \rangle$

lemma *K-statefun-comp* [*simp*]: $(K\text{-statefun } c \circ f) = K\text{-statefun } c$
 $\langle \text{proof} \rangle$

lemma *K-statefun-cong* [*cong*]: $K\text{-statefun } c \ x = K\text{-statefun } c \ x$
 $\langle \text{proof} \rangle$

definition *lookup* :: $('v \Rightarrow 'a) \Rightarrow 'n \Rightarrow ('n \Rightarrow 'v) \Rightarrow 'a$
where $\text{lookup } \text{destr } n \ s = \text{destr } (s \ n)$

definition *update* ::
 $('v \Rightarrow 'a1) \Rightarrow ('a2 \Rightarrow 'v) \Rightarrow 'n \Rightarrow ('a1 \Rightarrow 'a2) \Rightarrow ('n \Rightarrow 'v) \Rightarrow ('n \Rightarrow 'v)$
where $\text{update } \text{destr } \text{constr } n \ f \ s = s(n := \text{constr } (f (\text{destr } (s \ n))))$

lemma *lookup-update-same*:
 $(\bigwedge v. \text{destr } (\text{constr } v) = v) \implies \text{lookup } \text{destr } n \ (\text{update } \text{destr } \text{constr } n \ f \ s) =$
 $f (\text{destr } (s \ n))$
 $\langle \text{proof} \rangle$

lemma *lookup-update-id-same*:
 $\text{lookup } \text{destr } n \ (\text{update } \text{destr}' \ \text{id } n \ (K\text{-statefun } (\text{lookup } \text{id } n \ s'))) \ s =$
 $\text{lookup } \text{destr } n \ s'$
 $\langle \text{proof} \rangle$

lemma *lookup-update-other*:
 $n \neq m \implies \text{lookup } \text{destr } n \ (\text{update } \text{destr}' \ \text{constr } m \ f \ s) = \text{lookup } \text{destr } n \ s$
 $\langle \text{proof} \rangle$

lemma *id-id-cancel*: $id (id x) = x$
 ⟨proof⟩

lemma *destr-contstr-comp-id*: $(\bigwedge v. destr (constr v) = v) \implies destr \circ constr = id$
 ⟨proof⟩

lemma *block-conj-cong*: $(P \wedge Q) = (P \wedge Q)$
 ⟨proof⟩

lemma *conj1-False*: $P \equiv False \implies (P \wedge Q) \equiv False$
 ⟨proof⟩

lemma *conj2-False*: $Q \equiv False \implies (P \wedge Q) \equiv False$
 ⟨proof⟩

lemma *conj-True*: $P \equiv True \implies Q \equiv True \implies (P \wedge Q) \equiv True$
 ⟨proof⟩

lemma *conj-cong*: $P \equiv P' \implies Q \equiv Q' \implies (P \wedge Q) \equiv (P' \wedge Q')$
 ⟨proof⟩

lemma *update-apply*: $(update destr constr n f s x) =$
 $(if x=n then constr (f (destr (s n))) else s x)$
 ⟨proof⟩

lemma *ex-id*: $\exists x. id x = y$
 ⟨proof⟩

lemma *swap-ex-eq*:
 $\exists s. f s = x \equiv True \implies$
 $\exists s. x = f s \equiv True$
 ⟨proof⟩

lemmas *meta-ext = eq-reflection* [OF ext]

lemma *update d c n (K-statespace (lookup d n s)) s = s*
 ⟨proof⟩

end

4 Setup for State Space Locales

theory *StateSpaceLocale* **imports** *StateFun*
keywords *statespace* :: *thy-defn*
begin

$\langle ML \rangle$

For every type that is to be stored in a state space, an instance of this locale is imported in order convert the abstract and concrete values.

```
locale project-inject =
  fixes project :: 'value  $\Rightarrow$  'a
  and inject :: 'a  $\Rightarrow$  'value
  assumes project-inject-cancel [statefun-simp]: project (inject x) = x
begin
```

```
lemma ex-project [statefun-simp]:  $\exists v. \text{project } v = x$ 
 $\langle \text{proof} \rangle$ 
```

```
lemma project-inject-comp-id [statefun-simp]: project  $\circ$  inject = id
 $\langle \text{proof} \rangle$ 
```

```
lemma project-inject-comp-cancel[statefun-simp]: f  $\circ$  project  $\circ$  inject = f
 $\langle \text{proof} \rangle$ 
```

end

end

5 Syntax for State Space Lookup and Update

```
theory StateSpaceSyntax
imports StateSpaceLocale
begin
```

The state space syntax is kept in an extra theory so that you can choose if you want to use it or not.

```
syntax
  -statespace-lookup :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  'a  $\Rightarrow$  'c ( $\hookrightarrow$  [60, 60] 60)
  -statespace-update :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  'a  $\Rightarrow$  'c  $\Rightarrow$  ('a  $\Rightarrow$  'b)
  -statespace-updates :: ('a  $\Rightarrow$  'b)  $\Rightarrow$  updbinds  $\Rightarrow$  ('a  $\Rightarrow$  'b) ( $\hookrightarrow$  [900, 0] 900)
```

```
translations
  -statespace-updates f (-updbinds b bs) ==
    -statespace-updates (-statespace-updates f b) bs
  s<x:=y> == -statespace-update s x y
```

$\langle ML \rangle$

end

6 Examples

```
theory StateSpaceEx
imports StateSpaceLocale StateSpaceSyntax
begin
```

Did you ever dream about records with multiple inheritance? Then you should definitely have a look at state spaces. They may be what you are dreaming of. Or at least almost ...

Isabelle allows to add new top-level commands to the system. Building on the locale infrastructure, we provide a command **statespace** like this:

```
statespace vars =
  n::nat
  b::bool

print-locale vars-namespace
print-locale vars-valuetypes
print-locale vars
```

This resembles a **record** definition, but introduces sophisticated locale infrastructure instead of HOL type schemes. The resulting context postulates two distinct names *n* and *b* and projection / injection functions that convert from abstract values to *nat* and *bool*. The logical content of the locale is:

```
locale vars' =
  fixes n::'name and b::'name
  assumes distinct [n, b]

  fixes project-nat::'value  $\Rightarrow$  nat and inject-nat::nat  $\Rightarrow$  'value
  assumes  $\bigwedge n. \text{project-nat } (\text{inject-nat } n) = n$ 

  fixes project-bool::'value  $\Rightarrow$  bool and inject-bool::bool  $\Rightarrow$  'value
  assumes  $\bigwedge b. \text{project-bool } (\text{inject-bool } b) = b$ 
```

The HOL predicate *distinct* describes distinctness of all names in the context. Locale *vars'* defines the raw logical content that is defined in the state space locale. We also maintain non-logical context information to support the user:

- Syntax for state lookup and updates that automatically inserts the corresponding projection and injection functions.
- Setup for the proof tools that exploit the distinctness information and the cancellation of projections and injections in deductions and simplifications.

This extra-logical information is added to the locale in form of declarations, which associate the name of a variable to the corresponding projection and injection functions to handle the syntax transformations, and a link from the variable name to the corresponding distinctness theorem. As state spaces are merged or extended there are multiple distinctness theorems in the context. Our declarations take care that the link always points to the strongest distinctness assumption. With these declarations in place, a lookup can be written as $s \cdot n$, which is translated to $project\text{-}nat\ (s\ n)$, and an update as $s \langle n := 2 \rangle$, which is translated to $s(n := inject\text{-}nat\ 2)$. We can now establish the following lemma:

lemma (in *vars*) *foo*: $s \langle n := 2 \rangle \cdot b = s \cdot b$ *<proof>*

Here the simplifier was able to refer to distinctness of b and n to solve the equation. The resulting lemma is also recorded in locale *vars* for later use and is automatically propagated to all its interpretations. Here is another example:

statespace *'a varsX* = *NB*: *vars* [$n=N$, $b=B$] + *vars* + *x::'a*

The state space *varsX* imports two copies of the state space *vars*, where one has the variables renamed to upper-case letters, and adds another variable x of type *'a*. This type is fixed inside the state space but may get instantiated later on, analogous to type parameters of an ML-functor. The distinctness assumption is now *distinct* [N , B , n , b , x], from this we can derive both *distinct* [N , B] and *distinct* [n , b], the distinction assumptions for the two versions of locale *vars* above. Moreover we have all necessary projection and injection assumptions available. These assumptions together allow us to establish state space *varsX* as an interpretation of both instances of locale *vars*. Hence we inherit both variants of theorem *foo*: $s \langle N := 2 \rangle \cdot B = s \cdot B$ as well as $s \langle n := 2 \rangle \cdot b = s \cdot b$. These are immediate consequences of the locale interpretation action.

The declarations for syntax and the distinctness theorems also observe the morphisms generated by the locale package due to the renaming $n = N$:

lemma (in *varsX*) *foo*: $s \langle N := 2 \rangle \cdot x = s \cdot x$ *<proof>*

To assure scalability towards many distinct names, the distinctness predicate is refined to operate on balanced trees. Thus we get logarithmic certificates for the distinctness of two names by the distinctness of the paths in the tree. Asked for the distinctness of two names, our tool produces the paths of the variables in the tree (this is implemented in Isabelle/ML, outside the logic) and returns a certificate corresponding to the different paths. Merging state spaces requires to prove that the combined distinctness assumption implies the distinctness assumptions of the components. Such a proof is of the order

$m \cdot \log n$, where n and m are the number of nodes in the larger and smaller tree, respectively.

We continue with more examples.

```
statespace 'a foo =
  f::nat⇒nat
  a::int
  b::nat
  c::'a
```

```
lemma (in foo) foo1:
  shows s⟨a := i⟩.a = i
  ⟨proof⟩
```

```
lemma (in foo) foo2:
  shows (s⟨a:=i⟩).a = i
  ⟨proof⟩
```

```
lemma (in foo) foo3:
  shows (s⟨a:=i⟩).b = s.b
  ⟨proof⟩
```

```
lemma (in foo) foo4:
  shows (s⟨a:=i,b:=j,c:=k,a:=x⟩) = (s⟨b:=j,c:=k,a:=x⟩)
  ⟨proof⟩
```

```
statespace bar =
  b::bool
  c::string
```

```
lemma (in bar) bar1:
  shows (s⟨b:=True⟩).c = s.c
  ⟨proof⟩
```

You can define a derived state space by inheriting existing state spaces, renaming of components if you like, and by declaring new components.

```
statespace ('a,'b) loo = 'a foo + bar [b=B,c=C] +
  X::'b
```

```
lemma (in loo) loo1:
  shows s⟨a:=i⟩.B = s.B
  ⟨proof⟩
```

```
statespace 'a dup = FA: 'a foo [f=F, a=A] + 'a foo +
  x::int
```

```

lemma (in dup)
shows  $s < a := i > \cdot x = s \cdot x$ 
   $\langle proof \rangle$ 

```

```

lemma (in dup)
shows  $s < A := i > \cdot a = s \cdot a$ 
   $\langle proof \rangle$ 

```

```

lemma (in dup)
shows  $s < A := i > \cdot x = s \cdot x$ 
   $\langle proof \rangle$ 

```

There were known problems with syntax-declarations. They only worked when the context is already completely built. This is now overcome. e.g.:

```

locale fooX = foo +
assumes  $s < a := i > \cdot b = k$ 

```

We can also put statespaces side-by-side by using ordinary **locale** expressions (instead of the **statespace**).

```

locale side-by-side = foo + bar where  $b = B :: 'a$  and  $c = C$  for  $B\ C$ 

```

```

context side-by-side
begin

```

Simplification within one of the statespaces works as expected.

```

lemma  $s < B := i > \cdot C = s \cdot C$ 
   $\langle proof \rangle$ 

```

```

lemma  $s < a := i > \cdot b = s \cdot b$ 
   $\langle proof \rangle$ 

```

In contrast to the statespace *loo* there is no 'inter' statespace distinctness between the names of *foo* and *bar*.

```

end

```

Sharing of names in side-by-side statespaces is also possible as long as they are mapped to the same type.

```

statespace vars1 =  $n :: nat\ m :: nat$ 
statespace vars2 =  $n :: nat\ k :: nat$ 

```

```

locale vars1-vars2 = vars1 + vars2

```

```

context vars1-vars2
begin

```

Note that the distinctness theorem for *vars1* is selected here to do the proof.

```

lemma  $s < n := i > \cdot m = s \cdot m$ 

```

$\langle proof \rangle$

Note that the distinctness theorem for *vars2* is selected here to do the proof.

lemma $s < n := i > \cdot k = s \cdot k$

$\langle proof \rangle$

Still there is no inter-statespace distinctness.

lemma $s < k := i > \cdot m = s \cdot m$

$\langle proof \rangle$

end

statespace $merge\text{-}vars1\text{-}vars2 = vars1 + vars2$

context $merge\text{-}vars1\text{-}vars2$

begin

When defining a statespace instead of a side-by-side locale we get the distinctness of all variables.

lemma $s < k := i > \cdot m = s \cdot m$

$\langle proof \rangle$

end

6.1 Benchmarks

Here are some bigger examples for benchmarking.

$\langle ML \rangle$

0.2s

statespace $benchmark100 = A1::nat\ A2::nat\ A3::nat\ A4::nat\ A5::nat\ A6::nat\ A7::nat\ A8::nat\ A9::nat\ A10::nat\ A11::nat\ A12::nat\ A13::nat\ A14::nat\ A15::nat\ A16::nat\ A17::nat\ A18::nat\ A19::nat\ A20::nat\ A21::nat\ A22::nat\ A23::nat\ A24::nat\ A25::nat\ A26::nat\ A27::nat\ A28::nat\ A29::nat\ A30::nat\ A31::nat\ A32::nat\ A33::nat\ A34::nat\ A35::nat\ A36::nat\ A37::nat\ A38::nat\ A39::nat\ A40::nat\ A41::nat\ A42::nat\ A43::nat\ A44::nat\ A45::nat\ A46::nat\ A47::nat\ A48::nat\ A49::nat\ A50::nat\ A51::nat\ A52::nat\ A53::nat\ A54::nat\ A55::nat\ A56::nat\ A57::nat\ A58::nat\ A59::nat\ A60::nat\ A61::nat\ A62::nat\ A63::nat\ A64::nat\ A65::nat\ A66::nat\ A67::nat\ A68::nat\ A69::nat\ A70::nat\ A71::nat\ A72::nat\ A73::nat\ A74::nat\ A75::nat\ A76::nat\ A77::nat\ A78::nat\ A79::nat\ A80::nat\ A81::nat\ A82::nat\ A83::nat\ A84::nat\ A85::nat\ A86::nat\ A87::nat\ A88::nat\ A89::nat\ A90::nat\ A91::nat\ A92::nat\ A93::nat\ A94::nat\ A95::nat\ A96::nat\ A97::nat\ A98::nat\ A99::nat\ A100::nat$

2.4s

statespace $benchmark500 = A1::nat\ A2::nat\ A3::nat\ A4::nat\ A5::nat\ A6::nat\ A7::nat\ A8::nat\ A9::nat\ A10::nat\ A11::nat\ A12::nat\ A13::nat$

*A14::nat A15::nat A16::nat A17::nat A18::nat A19::nat A20::nat
A21::nat A22::nat A23::nat A24::nat A25::nat A26::nat A27::nat
A28::nat A29::nat A30::nat A31::nat A32::nat A33::nat A34::nat
A35::nat A36::nat A37::nat A38::nat A39::nat A40::nat A41::nat
A42::nat A43::nat A44::nat A45::nat A46::nat A47::nat A48::nat
A49::nat A50::nat A51::nat A52::nat A53::nat A54::nat A55::nat
A56::nat A57::nat A58::nat A59::nat A60::nat A61::nat A62::nat
A63::nat A64::nat A65::nat A66::nat A67::nat A68::nat A69::nat
A70::nat A71::nat A72::nat A73::nat A74::nat A75::nat A76::nat
A77::nat A78::nat A79::nat A80::nat A81::nat A82::nat A83::nat
A84::nat A85::nat A86::nat A87::nat A88::nat A89::nat A90::nat
A91::nat A92::nat A93::nat A94::nat A95::nat A96::nat A97::nat
A98::nat A99::nat A100::nat A101::nat A102::nat A103::nat A104::nat
A105::nat A106::nat A107::nat A108::nat A109::nat A110::nat A111::nat
A112::nat A113::nat A114::nat A115::nat A116::nat A117::nat A118::nat
A119::nat A120::nat A121::nat A122::nat A123::nat A124::nat A125::nat
A126::nat A127::nat A128::nat A129::nat A130::nat A131::nat A132::nat
A133::nat A134::nat A135::nat A136::nat A137::nat A138::nat A139::nat
A140::nat A141::nat A142::nat A143::nat A144::nat A145::nat A146::nat
A147::nat A148::nat A149::nat A150::nat A151::nat A152::nat A153::nat
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A161::nat A162::nat A163::nat A164::nat A165::nat A166::nat A167::nat
A168::nat A169::nat A170::nat A171::nat A172::nat A173::nat A174::nat
A175::nat A176::nat A177::nat A178::nat A179::nat A180::nat A181::nat
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A189::nat A190::nat A191::nat A192::nat A193::nat A194::nat A195::nat
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A203::nat A204::nat A205::nat A206::nat A207::nat A208::nat A209::nat
A210::nat A211::nat A212::nat A213::nat A214::nat A215::nat A216::nat
A217::nat A218::nat A219::nat A220::nat A221::nat A222::nat A223::nat
A224::nat A225::nat A226::nat A227::nat A228::nat A229::nat A230::nat
A231::nat A232::nat A233::nat A234::nat A235::nat A236::nat A237::nat
A238::nat A239::nat A240::nat A241::nat A242::nat A243::nat A244::nat
A245::nat A246::nat A247::nat A248::nat A249::nat A250::nat A251::nat
A252::nat A253::nat A254::nat A255::nat A256::nat A257::nat A258::nat
A259::nat A260::nat A261::nat A262::nat A263::nat A264::nat A265::nat
A266::nat A267::nat A268::nat A269::nat A270::nat A271::nat A272::nat
A273::nat A274::nat A275::nat A276::nat A277::nat A278::nat A279::nat
A280::nat A281::nat A282::nat A283::nat A284::nat A285::nat A286::nat
A287::nat A288::nat A289::nat A290::nat A291::nat A292::nat A293::nat
A294::nat A295::nat A296::nat A297::nat A298::nat A299::nat A300::nat
A301::nat A302::nat A303::nat A304::nat A305::nat A306::nat A307::nat
A308::nat A309::nat A310::nat A311::nat A312::nat A313::nat A314::nat
A315::nat A316::nat A317::nat A318::nat A319::nat A320::nat A321::nat
A322::nat A323::nat A324::nat A325::nat A326::nat A327::nat A328::nat
A329::nat A330::nat A331::nat A332::nat A333::nat A334::nat A335::nat
A336::nat A337::nat A338::nat A339::nat A340::nat A341::nat A342::nat
A343::nat A344::nat A345::nat A346::nat A347::nat A348::nat A349::nat
A350::nat A351::nat A352::nat A353::nat A354::nat A355::nat A356::nat*

*A357::nat A358::nat A359::nat A360::nat A361::nat A362::nat A363::nat
A364::nat A365::nat A366::nat A367::nat A368::nat A369::nat A370::nat
A371::nat A372::nat A373::nat A374::nat A375::nat A376::nat A377::nat
A378::nat A379::nat A380::nat A381::nat A382::nat A383::nat A384::nat
A385::nat A386::nat A387::nat A388::nat A389::nat A390::nat A391::nat
A392::nat A393::nat A394::nat A395::nat A396::nat A397::nat A398::nat
A399::nat A400::nat A401::nat A402::nat A403::nat A404::nat A405::nat
A406::nat A407::nat A408::nat A409::nat A410::nat A411::nat A412::nat
A413::nat A414::nat A415::nat A416::nat A417::nat A418::nat A419::nat
A420::nat A421::nat A422::nat A423::nat A424::nat A425::nat A426::nat
A427::nat A428::nat A429::nat A430::nat A431::nat A432::nat A433::nat
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A455::nat A456::nat A457::nat A458::nat A459::nat A460::nat A461::nat
A462::nat A463::nat A464::nat A465::nat A466::nat A467::nat A468::nat
A469::nat A470::nat A471::nat A472::nat A473::nat A474::nat A475::nat
A476::nat A477::nat A478::nat A479::nat A480::nat A481::nat A482::nat
A483::nat A484::nat A485::nat A486::nat A487::nat A488::nat A489::nat
A490::nat A491::nat A492::nat A493::nat A494::nat A495::nat A496::nat
A497::nat A498::nat A499::nat A500::nat*

9.0s

statespace *benchmark1000 = A1::nat A2::nat A3::nat A4::nat A5::nat
A6::nat A7::nat A8::nat A9::nat A10::nat A11::nat A12::nat A13::nat
A14::nat A15::nat A16::nat A17::nat A18::nat A19::nat A20::nat
A21::nat A22::nat A23::nat A24::nat A25::nat A26::nat A27::nat
A28::nat A29::nat A30::nat A31::nat A32::nat A33::nat A34::nat
A35::nat A36::nat A37::nat A38::nat A39::nat A40::nat A41::nat
A42::nat A43::nat A44::nat A45::nat A46::nat A47::nat A48::nat
A49::nat A50::nat A51::nat A52::nat A53::nat A54::nat A55::nat
A56::nat A57::nat A58::nat A59::nat A60::nat A61::nat A62::nat
A63::nat A64::nat A65::nat A66::nat A67::nat A68::nat A69::nat
A70::nat A71::nat A72::nat A73::nat A74::nat A75::nat A76::nat
A77::nat A78::nat A79::nat A80::nat A81::nat A82::nat A83::nat
A84::nat A85::nat A86::nat A87::nat A88::nat A89::nat A90::nat
A91::nat A92::nat A93::nat A94::nat A95::nat A96::nat A97::nat
A98::nat A99::nat A100::nat A101::nat A102::nat A103::nat A104::nat
A105::nat A106::nat A107::nat A108::nat A109::nat A110::nat A111::nat
A112::nat A113::nat A114::nat A115::nat A116::nat A117::nat A118::nat
A119::nat A120::nat A121::nat A122::nat A123::nat A124::nat A125::nat
A126::nat A127::nat A128::nat A129::nat A130::nat A131::nat A132::nat
A133::nat A134::nat A135::nat A136::nat A137::nat A138::nat A139::nat
A140::nat A141::nat A142::nat A143::nat A144::nat A145::nat A146::nat
A147::nat A148::nat A149::nat A150::nat A151::nat A152::nat A153::nat
A154::nat A155::nat A156::nat A157::nat A158::nat A159::nat A160::nat
A161::nat A162::nat A163::nat A164::nat A165::nat A166::nat A167::nat
A168::nat A169::nat A170::nat A171::nat A172::nat A173::nat A174::nat
A175::nat A176::nat A177::nat A178::nat A179::nat A180::nat A181::nat*

[illegible]

[illegible]

A868::nat A869::nat A870::nat A871::nat A872::nat A873::nat A874::nat
A875::nat A876::nat A877::nat A878::nat A879::nat A880::nat A881::nat
A882::nat A883::nat A884::nat A885::nat A886::nat A887::nat A888::nat
A889::nat A890::nat A891::nat A892::nat A893::nat A894::nat A895::nat
A896::nat A897::nat A898::nat A899::nat A900::nat A901::nat A902::nat
A903::nat A904::nat A905::nat A906::nat A907::nat A908::nat A909::nat
A910::nat A911::nat A912::nat A913::nat A914::nat A915::nat A916::nat
A917::nat A918::nat A919::nat A920::nat A921::nat A922::nat A923::nat
A924::nat A925::nat A926::nat A927::nat A928::nat A929::nat A930::nat
A931::nat A932::nat A933::nat A934::nat A935::nat A936::nat A937::nat
A938::nat A939::nat A940::nat A941::nat A942::nat A943::nat A944::nat
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A952::nat A953::nat A954::nat A955::nat A956::nat A957::nat A958::nat
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A966::nat A967::nat A968::nat A969::nat A970::nat A971::nat A972::nat
A973::nat A974::nat A975::nat A976::nat A977::nat A978::nat A979::nat
A980::nat A981::nat A982::nat A983::nat A984::nat A985::nat A986::nat
A987::nat A988::nat A989::nat A990::nat A991::nat A992::nat A993::nat
A994::nat A995::nat A996::nat A997::nat A998::nat A999::nat A1000::nat

lemma (in *benchmark100*) test: $s \langle A1 := a \rangle \cdot A100 = s \cdot A100$ *<proof>*
lemma (in *benchmark500*) test: $s \langle A1 := a \rangle \cdot A100 = s \cdot A100$ *<proof>*
lemma (in *benchmark1000*) test: $s \langle A1 := a \rangle \cdot A100 = s \cdot A100$ *<proof>*

end