

## Ordinal Numbers

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**Definition 0.1.** An ordinal number is a transitive set  $\alpha$  such that every element of  $\alpha$  is a transitive set.

Let an ordinal stand for an ordinal number.

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**Definition 0.2.** **Ord** is the class of all ordinals.

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**Proposition 0.3.** Let  $\alpha$  be an ordinal. Then every element of  $\alpha$  is an ordinal.

*Proof.* Let  $x$  be an element of  $\alpha$ . Then  $x$  is transitive.

Let us show that every element of  $x$  is a subset of  $x$ . Let  $y$  be an element of  $x$ . Then  $y$  is a subset of  $x$ . Let  $z$  be an element of  $y$ . Every element of  $y$  is an element of  $x$ . Hence  $z$  is an element of  $x$ . End.

Thus every element of  $x$  is transitive. Therefore  $x$  is an ordinal.  $\square$