

The Natural Numbers Form a Set

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Proposition 0.1. \mathbb{N} is a set.

Proof. Define $f(n) = n + 1$ for $n \in \mathbb{N}$. Then f is a map from \mathbb{N} to \mathbb{N} . Hence we can take a subset X of \mathbb{N} that is inductive regarding 0 and f . Then $0 \in X$ and for all $n \in X$ we have $n + 1 \in X$. Hence X contains every natural number. Thus we have $\mathbb{N} \subseteq X$ and $X \subseteq \mathbb{N}$. Therefore $\mathbb{N} = X$. Consequently \mathbb{N} is a set. \square