

Axiom 1 (Foundation). Let X be a nonempty system of sets. Then X has an element x such that X and x are disjoint.

Corollary 2. Let X be a nonempty system of sets. Then X has an element x such that for no $y \in X$ we have $y \in x$.

Proposition 3. Let Φ be a class. Assume that for all sets x if Φ contains every element of x that is a set then Φ contains x . Then Φ contains every set.

Proof. Assume the contrary. Define $M := \{x \mid x \text{ is a set such that } x \notin \Phi\}$. Then M is nonempty. Hence we can take a $x \in M$ such that for no $y \in M$ we have $y \in x$. Then x is a set such that every element of x that is a set is contained in Φ . Thus Φ contains x . Contradiction. ■