Burali-Forti's Paradox in ℕaproche

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Burali-Forti's Paradox, named after Cesare Burali-Forti, demonstates that the collection $\mathbb{O}$ of all ordinal numbers is a proper class [1].

Theorem 1 (Burali-Forti's Paradox). $\mathbb{O}$ is not a set.

Proof. Assume the contrary. $\mathbb{O}$ is transitive and every element of $\mathbb{O}$ is transitive. Hence $\mathbb{O}$ is an ordinal. Thus $\mathbb{O} \in \mathbb{O}$. Contradiction. ■

References

[1] Cesare Burali-Forti. "Una questione sui numeri transfiniti". In: *Rendiconti del Circolo Matematico di Palermo* 11 (1897), pp. 154–164. DOI: 10.1007/BF03015911.

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