Cantor's Theorem in ℕaproche

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2025

This is a formalization of *Cantor's Theorem* [1], i.e. of the assertion that any set x is strictly smaller than its powerset $\mathcal{P}(x)$:

Theorem 1 (Cantor's Theorem). Let x be a set. Then $|x| < |\mathcal{P}(x)|$.

Proof.

Case x is empty. □

Case x is nonempty. Assume the contrary. Then $|x| \geq |\mathcal{P}(x)|$. Hence there exists a surjective map from x onto $\mathcal{P}(x)$ (by existence condition for surjections). Indeed $\mathcal{P}(x)$ is a nonempty set. Contradiction (by Cantor (elementary version)). □

■

References

[1] Georg Cantor. "Über eine elementare Frage der Mannigfaltigskeitslehre". In: *Jahresbericht der Deutschen Mathematiker-Vereinigung* 1 (1891), pp. 75–78.

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