The Russell-Myhill Paradox in Naproche

Marcel Schütz

2025

The *Russell-Myhill Paradox* is a paradox discovered by Bertrand Russell which states that two systems of propositions with the same *logical product* are not necessarily equal. It was discussed in Russell's 1903 *Principles of Mathematics* [2, Appendix B] and rediscovered in 1958 by John Myhill [1].

Signature. A *proposition* is an object.

Signature. Let p be a proposition. p is *true* is an atom.

Definition. A *system of propositions* is a class P such that every element of P is a proposition.

Signature. Let P be a system of propositions. The *logical product of* P is a proposition p such that p is true iff every element of P is true. Let $\sqcap P$ denote the logical product of P.

Theorem (Russell-Myhill Paradox). It is wrong that

$$\sqcap P = \sqcap Q \Longrightarrow P = Q$$

for all systems of propositions P, Q.

Proof. Assume the contrary. Define

$$Q := \left\{ q \;\middle|\; \begin{array}{l}\text{there exists a system of propositions } P \text{ such that } q = \sqcap P \text{ and} \\ q \notin P\end{array} \right\}.$$

Consider $q = \sqcap Q$. Then for any system of propositions P such that $\sqcap P = q$ we have $P = Q$. Hence $q \in Q$ iff $q \notin Q$. Contradiction. ■

References

[1] John Myhill. "Problems Arising in the Formalization of Intensional Logic". In: *Logique Et Analyse* 1.1 (1958), pp. 78–83. DOI: 10.2307/2272577.

[2] Bertrand Russell. *The Principles of Mathematics*. Cambridge University Press, 1903.

License

© Marcel Schütz (2024–2025). This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 4.0 International" (CC BY-NC-SA 4.0) license.