Russell's Paradox in ℕaproche

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Russell's Paradox is a set-theoretical paradox discovered by Bertrand Russell around 1902 [1, chapter XV] which shows that there exist statements φ whose extension $\{x \mid \varphi(x)\}$ cannot constitute a set – or in other words: Not every class is a set.

Theorem 1 (Russell's Paradox). There exists a class that is not a set.

Proof. Assume the contrary. Define $R := \{x \mid x \text{ is a set such that } x \notin x\}$. Then R is a set. Hence $R \in R$ iff $R \notin R$. Contradiction. ■

References

[1] Gottlob Frege. *The Philosophical and Mathematical Correspondence*. Ed. by Gottfried Gabriel et al. Basil Blackwell, 1980.

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