Definition 1. Let n be a natural number and m be a nonzero divisor of n. $\frac{n}{m}$ is the natural number k such that $k \cdot m = n$. Let the *quotient of n and m* stand for $\frac{n}{m}$.

Proposition 2. Let n be a natural number. $\frac{n}{1} = n$.

Proof. We have $\frac{n}{1} = \frac{n}{1} \cdot 1 = n$. ■

Proposition 3. Let n be a natural number and m be a nonzero divisor of n. Then $\frac{n}{m} = 0$ iff $n = 0$.

Proof.

Case $\frac{n}{m} = 0$. Then $n = \frac{n}{m} \cdot m = 0 \cdot m = 0$. □

Case $n = 0$. Then $\frac{n}{m} \cdot m = n = 0$. Hence $\frac{n}{m} = 0$ or $m = 0$. m is nonzero. Thus $\frac{n}{m} = 0$. □

■

Proposition 4. Let n, m, k be natural numbers such that k is nonzero. Assume $k \mid n, m$. Then

$$\frac{n+m}{k} = \frac{n}{k} + \frac{m}{k}.$$

Proof. We have $\frac{n+m}{k} \cdot k = n + m$ and $\frac{n}{k} \cdot k = n$ and $\frac{m}{k} \cdot k = m$. Hence

$$\frac{n+m}{k} \cdot k = \left(\frac{n}{k} \cdot k\right) + \left(\frac{m}{k} \cdot k\right) = \left(\frac{n}{k} + \frac{m}{k}\right) \cdot k.$$

Thus $\frac{n+m}{k} = \frac{n}{k} + \frac{m}{k}$. ■

Proposition 5. Let n, m be natural numbers and k be a nonzero divisor

of m. Then

$$\frac{n \cdot m}{k} = n \cdot \frac{m}{k}.$$

Proof. We have $\frac{n \cdot m}{k} \cdot k = n \cdot m$ and $\frac{m}{k} \cdot k = m$. Hence

$$\frac{n \cdot m}{k} \cdot k = n \cdot \left(\frac{m}{k} \cdot k\right) = \left(n \cdot \frac{m}{k}\right) \cdot k.$$

Thus $\frac{n \cdot m}{k} = n \cdot \frac{m}{k}$. ■

Corollary 6. Let n, m be natural numbers and k be a nonzero divisor of m. Then

$$\frac{n \cdot m}{k} = \frac{m}{k} \cdot n.$$

Corollary 7. Let n, m be natural numbers and k be a nonzero divisor of n. Then

$$\frac{n \cdot m}{k} = \frac{n}{k} \cdot m.$$

Corollary 8. Let n, m be natural numbers and k be a nonzero divisor of n. Then

$$\frac{n \cdot m}{k} = m \cdot \frac{n}{k}.$$

Proposition 9. Let n, k be natural numbers such that k be nonzero and m be a nonzero divisor of n. Then

$$\frac{n \cdot k}{m \cdot k} = \frac{n}{m}.$$

Proof. We have $\frac{n \cdot k}{m \cdot k} \cdot (m \cdot k) = n \cdot k$. Hence

$$\left(\frac{n \cdot k}{m \cdot k} \cdot m\right) \cdot k = \frac{n \cdot k}{m \cdot k} \cdot (m \cdot k) = n \cdot k.$$

Thus $\frac{n \cdot k}{m \cdot k} \cdot m = n$. Therefore $\frac{n}{m} = \frac{n \cdot k}{m \cdot k}$. ■

Corollary 10. Let n, k be natural numbers such that k be nonzero and m be a nonzero divisor of n. Then

$$\frac{k \cdot n}{k \cdot m} = \frac{n}{m}.$$