Definition 1. A *system of sets* is a class X such that every element of X is a set.

Definition 2. A *system of nonempty sets* is a class X such that every element of X is a nonempty set.

Definition 3. Let A be a class. A *system of subsets of A* is a class X such that every element of X is a subset of A.

Proposition 4. Let A be a class. Then $\emptyset$ is a system of subsets of A.

Proposition 5. Let A be a class. Then $\mathcal{P}(A)$ is a system of subsets of A.

Proposition 6. Let X, Y be systems of sets. Then $X \cup Y$ is a system of sets.

Proposition 7. Let X, Y be systems of sets. Then $X \cap Y$ is a system of sets.

Proposition 8. Let X, Y be systems of sets. Then $X \setminus Y$ is a system of sets.

Part I
Unions Over Systems of Sets

Definition 9. Let X be a system of sets. $\bigcup X := \{a \mid a \in x \text{ for some } x \in X\}$. Let *union over X* stand for $\bigcup X$.

Proposition 10. $\bigcup \emptyset = \emptyset$.

Proof. $\bigcup \emptyset = \{a \mid a \in x \text{ for some } x \in \emptyset\}$. $\emptyset$ has no elements. Hence there is no object a such that $a \in x$ for some $x \in \emptyset$. Thus $\bigcup \emptyset = \emptyset$. ■

Proposition 11. Let x, y be sets. Then $\bigcup \{x,y\} = x \cup y$.

Proof. Let us show that $\bigcup \{x,y\} \subset x \cup y$. Let $a \in \bigcup \{x,y\}$. Then a is contained in some element of $\{x,y\}$. Hence $a \in x$ or $a \in y$. Thus $a \in x \cup y$. End.

Let us show that $x \cup y \subset \bigcup \{x,y\}$. Let $a \in x \cup y$. Then $a \in x$ or $a \in y$. Hence a is contained in some element of $\{x,y\}$. Therefore $a \in \bigcup \{x,y\}$. End. ■

Corollary 12. Let x be a set. Then $\bigcup \{x\} = x$.

Part II
Intersections Over Systems of Sets

Definition 13. Let X be a system of sets. $\bigcap X := \{a \mid a \in x \text{ for all } x \in X\}$. Let the *intersection over X* stand for $\bigcap X$.

Proposition 14. $\bigcap \emptyset$ is the class of all objects.

Proof. Define $V := \{x \mid x \text{ is an object}\}$. We have $\bigcap \emptyset \subset V$. Indeed every element of $\bigcap \emptyset$ is an object.

Let us show that $V \subset \bigcap \emptyset$. Let $a \in V$. Then a is an object. For every $x \in \emptyset$ we have $a \in x$. Indeed $\emptyset$ has no elements. Thus $a \in \bigcap \emptyset$. End. ■

Proposition 15. Let x, y be sets. Then $\bigcap\{x,y\} = x \cap y$.

Proof. Let us show that $\bigcap\{x,y\} \subset x \cap y$. Let $a \in \bigcap\{x,y\}$. Then a is contained in every element of $\{x,y\}$. Hence $a \in x$ and $a \in y$. Thus $a \in x \cap y$. End.

Let us show that $x \cap y \subset \bigcap\{x,y\}$. Let $a \in x \cap y$. Then $a \in x$ and $a \in y$. Hence a is contained in every element of $\{x,y\}$. Therefore $a \in \bigcap\{x,y\}$. End. ■

Corollary 16. Let x be a set. Then $\bigcap\{x\} = x$.