Definition 1. Let x be a set. x is *finite* iff $|x| < \omega$.

Definition 2. Let x be a set. x is *infinite* iff x is not finite.

Proposition 3. Let x be a set. Then x is finite iff $|x| = n$ for some $n \in \omega$.

Proposition 4. Let x be a set. Then x is infinite iff $|x| \geq \omega$.

Proof. $|x| \geq \omega$ iff $|x| \not< \omega$. ■

Proposition 5. ω is infinite.