**Lemma 1.** For every ordinal $\alpha$ there exists a cardinal greater than $\alpha$.

*Proof.* Let $\alpha$ be an ordinal. Take $\kappa = |\mathcal{P}(\alpha)|$. Then $\kappa > |\alpha|$.

Let us show that $\kappa > \alpha$. Assume the contrary. Then $|\mathcal{P}(\alpha)| = \kappa \leq \alpha$. Hence $\kappa = |\mathcal{P}(\alpha)| = ||\mathcal{P}(\alpha)|| \leq |\alpha|$. Contradiction. End. ■

**Definition 2.** Let $\kappa$ be a cardinal. $\kappa^+$ is the cardinal such that $\kappa < \kappa^+$ and there is no cardinal $\nu$ such that $\kappa < \nu < \kappa^+$.

**Definition 3.** A *successor cardinal* is a cardinal number $\kappa$ such that $\kappa = \nu^+$ for some cardinal number $\nu$.

**Proposition 4.** Let $\kappa$ be a cardinal. Then $|\alpha| \leq \kappa$ for every $\alpha \in \kappa^+$.