Proposition 1. Let a, b be distinct objects. Then $|\{a,b\}| = 2$.

Proof. Define

$$f(x) := \begin{cases} 0 & x = a \\ 1 & x = b \end{cases}$$

for $x \in \{a,b\}$. Then f is a map from $\{a,b\}$ to 2. f is injective and surjective onto 2. Hence f is a bijection between $\{a,b\}$ and 2. Consequently $\{a,b\}$ and 2 are equinumerous. Thus $|\{a,b\}| = |2| = 2$. ■